Non-Parallel Electric and Magnetic Fields in a Gravitational Background, Stationary Gravitational Waves and Gravitons

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Abstract

The existence of an electromagnetic field with parallel electric and magnetic components is readdressed in the presence of a gravitational field. A non-parallel solution is shown to exist. Next, we analyse the possibility of finding stationary gravitational waves in de nature. Finaly, We construct a D=4 effecttive quantum gravity model. Tree-level unitarity is verified.

1 Electric and Magnetic Field in a Gravitational Background

Based on a series of papers by Brownstein [1] and Salingaros [2], we readdress here the possibility of the existence of an electromagnetic field whose electric and magnetic components are parallel in the presence of a gravitational field. The coupling between the electromagnetic sector and the gravitational background is accomplished by means of the action.

$$\mathcal{S} = \int \sqrt{-\tilde{g}}(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})d^4x, \tag{1.1}$$

where

$$\tilde{g} = det(g_{\mu\nu}),$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

From the above action, the following field-equations follow:

$$\mathcal{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu} + \Gamma^{\beta}_{\beta\lambda}F^{\lambda\nu} + \Gamma^{\nu}_{\mu\lambda}F^{\mu\lambda} = J^\nu \tag{1.2}$$

$$\mathcal{D}_\mu F_{\nu\beta} + \mathcal{D}_\nu F_{\beta\mu} + \mathcal{D}_\beta F_{\mu\nu} = 0. \tag{1.3}$$

Choosing the background to be described by the F.R.W metric,

$$d\mathrm{S}^2 = dt^2 - a^2(t)\Big[\frac{dr^2}{1-Ar^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\Big], \tag{1.4}$$

the Maxwell equations in the absence of electromagnetic sources read as below:

$$\begin{gathered}\vec{\nabla}\cdot\vec{E} = g\vec{\nabla}f\cdot\vec{E} \qquad (1.5)\\ \vec{\nabla}\cdot\vec{B} = 0\\ \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\\ \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} - g\frac{\partial f}{\partial t}\vec{E} + g\vec{\nabla}f\times\vec{B} - \Gamma^{i}_{\mu\beta}F^{\mu\beta},\end{gathered}$$

where

$$g = \frac{\sqrt{1 - Ar^2}}{a^3 r^2 \sin\theta}, \qquad f = \frac{a^3 r^2 \sin\theta}{\sqrt{1 - Ar^2}}, \tag{1.6}$$

and $A = +1, 0, -1$.
The Wave-equations for $\vec{E}$ and $\vec{B}$ are found to be given by:

$$\nabla^2 \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{\nabla}(g\vec{\nabla} f \cdot \vec{E}) - \frac{\partial g}{\partial t}\frac{\partial f}{\partial t}\vec{E} - g\frac{\partial^2 f}{\partial t^2}\vec{E} - g\frac{\partial f}{\partial t}\frac{\partial \vec{E}}{\partial t} + \tag{1.7}$$

$$+\frac{\partial g}{\partial t}\vec{\nabla} f \times \vec{B} + g\frac{\partial}{\partial t}\vec{\nabla} f \times \vec{B} + g\vec{\nabla} f \times \frac{\partial \vec{B}}{\partial t} - \frac{\partial}{\partial t}(\Gamma^i_{\mu\beta} F^{\mu\beta});$$

$$\nabla^2 \vec{B} - \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{\nabla} \times (g\frac{\partial f}{\partial t}\vec{E} - g\vec{\nabla} f \times \vec{B} + \Gamma^i_{\mu\beta} F^{\mu\beta}) \tag{1.8}$$

Now, by virtue of the presence of the gravitational background, we have explicitly built up a solution for $\vec{E}$ end $\vec{B}$ that evade the claim by Brownstein [1] and Salingaros [2]. These authors state that it is always possible to find out parallel solutions for $\vec{E}$ and $\vec{B}$ in Plasma Physics or in an Astrophysic Plasma. However, contrary to their result, we have found non-parallel solutions due to the non-flat background of gravity:

$$\vec{E} = \hat{\imath}(\sin\theta G_{(r,t,\theta)} - Cos\theta F_{(r,t)})kaCos(kz)Cos(\omega t) + \tag{1.9}$$

$$+\hat{\jmath}(\cos\theta F_{(r,t)} - \sin\theta G_{(r,t,\theta)})ka\sin(kz)\cos(\omega t) +$$

$$+\hat{k}[(\sin\theta\cos\varphi F_{(r,t)} + \cos\theta\cos\varphi G_{(r,t,\theta)})ka\cos(kz)\cos(\omega t) +$$

$$-(\sin\theta\sin\varphi F_{(r,t)} + \cos\theta\sin\varphi G_{(r,t,\theta)})ka\sin(kz)\cos(\omega t)]$$

and

$$\vec{B} = ka[\hat{\imath}\sin(kz) + \hat{\jmath}Cos(kz)]Cos(\omega t) \tag{1.10}$$

where the functions $G_{(r,t,\theta)} = \frac{aCotg\theta}{3\dot{a}r}$ and

$$F_{(r,t)} = \frac{2a}{3\dot{a}r} + \frac{Aar}{3\dot{a}(1 - Ar^2)} \tag{1.11}$$

are the metric contribution.

2 Stationary Gravitational Waves and Gravitons

Now, we analyse the possibility of finding stationary gravitational waves. From a phenomenological viewpoint, a distribution of black holes could play the role of knots for the non-propagating gravitational waves. We postulate the equation that may lead to this sort of waves:

$$R_{\mu\nu} = \kappa\Lambda h_{\mu\nu}, \tag{2.12}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{2.13}$$

where Λ is the cosmological constant. These equations yield:

$$\partial_\beta\partial_\nu h^\beta_\mu + \partial_\beta\partial_\mu h^\beta_\nu - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h^\beta_\beta = \Lambda h_{\mu\nu}. \tag{2.14}$$

Now, solution of the form

$$h_{\mu\nu} = C_{\mu\nu}(z) f(t), \tag{2.15}$$

$$h_{\mu\nu} = \begin{pmatrix} A_{00} & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & A_{00} \end{pmatrix} e^{i\tilde{k}z} \cos\omega t, \tag{2.16}$$

can be found, where A_{00}, A_{11} and A_{12} are free parameters, whereas $\tilde{k} = \sqrt{\Lambda - \omega^2}$ is the wave number. Having in mind that Λ is a small number, the frequency ω must be extremely small. This forces on us to search for a mechanism to detect such low-frequency stationary waves.

The equations of motion stemming from 2.12 may be derived from the density Lagrangian

$$\mathcal{L}_H = \frac{1}{2}H^{\mu\nu}\Box H_{\mu\nu} - \frac{1}{4}H\Box H - \frac{1}{2}H^{\mu\nu}\partial_\mu\partial_\alpha H^\alpha_\nu - \frac{1}{2}H^{\mu\nu}\partial_\nu\partial_\alpha H^\alpha_\mu + \tag{2.17}$$
$$-\frac{1}{2}\Lambda H^{\mu\nu}H_{\mu\nu} + \frac{1}{4}\Lambda H^2,$$

where

$$H^{\alpha}_{\nu} = h^{\alpha}_{\nu} - \frac{1}{2}\delta^{\alpha}_{\nu} h$$

and the bilinear form correspondent operator of Lagrangian 2.17 is given by

$$\Theta_{\mu\nu,\kappa\Lambda} = (\Box - \Lambda)P^{(2)} - \Lambda P^{(1)}_{m} + \frac{5}{2}(\Box - \Lambda)P^{(0)}_{s} - \frac{(\Lambda + 3\Box)}{2}P^{(0)}_{w} + \tag{2.18}$$

$$+\frac{\sqrt{3}}{2}(\Lambda - \Box)P^{(0)}_{sw} + \frac{\sqrt{3}}{2}(\Lambda - \Box)P^{(0)}_{ws},$$

and $P^{(i)}$, i=0,1,2, are spin-projetor operators in the space of rank-2 symmetric tensors. The graviton propagator read off from this Lagrangian is given by:

$$\left\langle T(h_{\mu\nu}(x); h_{\kappa\lambda}(y))\right\rangle = i\Theta^{-1}_{\mu\nu,\kappa\lambda}\delta^{4}(x-y) \tag{2.19}$$

where

$$\Theta^{-1} = \left[XP^{(2)} + YP^{(1)}_{m} + ZP^{(0)}_{s} + WP^{(0)}_{w} + RP^{(0)}_{sw} + SP^{(0)}_{ws}\right]_{\mu\nu,\kappa\lambda} \tag{2.20}$$

with

$$X = -\frac{1}{\Lambda - \Box}; \qquad Y = -\frac{1}{\Lambda}; \qquad Z = -\frac{\Lambda + 3\Box}{\Lambda^{2} + 8\Lambda\Box - 9\Box^{2}}; \tag{2.21}$$

$$W = -\frac{5}{\Lambda - 9\Box}; \qquad R = -\frac{\sqrt{3}}{\Lambda + 9\Box}; \qquad S = -\frac{\sqrt{3}}{\Lambda + 9\Box};$$

From this propagator, we can set a current-current amplitude and discuss tree-level unitarity [3]. Three massive excitations were found: They are a spin-2 quantum with mass equal to $k^2 = \Lambda$ and two massive spin-0 quanta with masses equal to $k^2 = \Lambda$ and $k^2 = -\frac{1}{9}\Lambda$. The spin-2 is a physical one: the imaginary part of the residue of the amplitude at the pole $k^2 = \Lambda$ is positive, so that it does not lead to a ghost. It remains to be shown that the tachyonic pole, $k^2 = -\frac{1}{9}\Lambda$, is non-dynamical or decouples through some constraint on the sources.

We conclude, then, that in a gravitational background it is always possible to find non-parallel electric and magnetic fields. It is the gravitational field that breaks the parallel configuration of $\vec{E}$ and $\vec{B}$ argued by [1, 2]. Also, a stationary gravitational wave equation was postulated and a particular solution was found. We argue that such a kind of solution is possible to be found in Black Hole distributions. Finaly we set up an effective quantum gravity model where the necessary condition for the tree-level unitarity for the spin-2 sector is respected. The model is infrared finite though non-renormalizable in the ultravioled limit.

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